Homework 11 (Due 4/23/2014)

Math 622

April 8, 2014

The questions on this assignment cover the material and concepts of section 10.2. In reading this section, it is not necessary to read the derivation of the canonical two-factor model, from pages 407 to the middle of 411; you should read the bottom of page 406 and the very top of 407 and understand when the two-factor Vasicek model can be reduced to canonical form and understand the importance of this reduction. Study also the derivation of the zero-coupon bond price on pages 411-413. For now you can skip the sub-section, *Short Rates and Long Rates*, pages 414-416. However, you should read the sub-section, *Gaussian Factor Processes*; this is also treated in the class notes. Read the remaining part of section 10.2 on two-factor CIR and mixed models.

Hand in all problems.

1. The Hull-White interest model is defined in section 6.5. Read this section. You will see that the Hull-White model is also an affine-yield model and that one can find a formula for $B(t,T)$ by the same pde method we used in class for the two-factor Vasicek model (see also Shreve, pages 411-413).

a) Do Exercise 6.3, Shreve.

b) For the Hull-White model, as treated in Example 6.5.1, we would like to derive a stochastic differential equation model for the zero-coupon bond price itself. Using the results of Example 6.5.1 on the Hull-White model, show that $d_t[D(t)B(t,T)] = -\sigma D(t)C(t,T)B(t,T)\, d\widetilde{W}(t)$ for $t \leq T$.
(Apply Itô's rule; use equations (6.5.8) and (6.5.9).)

(c) Let $\widetilde{\mathbf{P}}^T$ be the risk-neutral measure when $B(t,T)$ is used as a numéraire; see section 9.4.3. Use the expression for $d_t[D(t)B(t,T)]$ obtained in part b) to construct a process $\widetilde{W}^T$ that is a Brownian motion under $\widetilde{\mathbf{P}}^T$. Let $dS(t) = R(t)S(t)\,dt + \gamma S(t)\,d\widetilde{W}(t)$ (γ is the volatility here since we have already used σ). Write a stochastic differential for the forward price, $\mathrm{For}_S(t,T)$, in terms of $d\widetilde{W}^T(t)$.

2. Shreve, Exercise 10.2

3. Shreve, Exercise 10.3

4. Shreve, Exercise 10.7